From equations 6.27 and 6.28,

$$p_D = k - t_x = q - k(1 + 2\alpha)$$

$$q + t_x = 2k(1 + \alpha)$$

Combining this with equation 6.29, for the chosen reduction of area,

$$\left.\begin{aligned} \frac{t_x}{2k} &= r(1 + \alpha) = \frac{2(1 + \alpha)\sin\alpha}{1 + 2\sin\alpha} \\ \frac{q}{2k} &= (1 + \alpha) - \frac{t_x}{2k} = \frac{1 + \alpha}{1 + 2\sin\alpha} \end{aligned}\right\} \quad (6.30)$$

These explicit values can be obtained for only one reduction, such that BD is straight. For 15° semi-angle, this reduction is 34%. Then

$$\frac{t_x}{2k} = r(1 + \alpha) = 0{\cdot}34\ (1 + 0{\cdot}262) = 0{\cdot}429$$

The stress required to produce homogeneous deformation to 34% is given by equation 6.3, making allowance for the increased yield stress in plane strain,

$$\frac{\sigma}{2k} = \ln\frac{1}{1 - r} = \ln\frac{1}{0{\cdot}66} = 0{\cdot}415$$

The contribution of redundant work is thus small, and can be neglected. If lower die angles are used the redundant work contribution at 34% reduction is even less. It is therefore of greater interest to consider the more complex slip-line field solution for lower reductions or higher die angles, where the redundant work is appreciable.

When the reduction of area is less than $2\sin\alpha/(1 + 2\sin\alpha)$, it is necessary to extend the field further, from the fans centred on A and B.

6.5.2 *The slip-line field for reductions less than* $2\sin\alpha/(1 + 2\sin\alpha)$. *Construction for extending slip-line fields from radial fans*

The field may conveniently be extended by a graphical method[6.4]. The basic arcs are divided into suitable equal intervals by radii from A and B, as in Figure 6.5. It is usual to choose 5° intervals, which give good accuracy, but the explanation can be given more simply in terms of a 10° net.

The slip-line BC_1 must rotate through 10° in passing from C_1 to the point 6. This is easily shown from the Hencky equations and is known as Henckys' 1st Theorem (see Example 5.3). The slip-line AC'_1 must similarly rotate through 10° between C'_1 and the point 6. The position of 6 is thus found accurately by drawing a line C_16 from C_1, at half this angle (5°) to the radius BC_1, to represent the chord of the arc C_16. A chord is similarly drawn from C'_1 at 5° to AC'_1. The intersection of these lines gives the position of 6. The next mesh is started with a line from C_2 at 5° to BC_2, showing the direction to 5. This intersects the chord starting at 6 in a direction at 10° to C'_1 6, because each chord at 6 will be at 5° to the tangent, giving the position of 5. The field is thus built up in unit cells. Finally, smooth orthogonal curves are drawn through the points of intersection. The accuracy is obviously improved if a gent to CB at C makes an angle $\left(\frac{\pi}{4} - \zeta + \psi\right)$, but CB is a straight line at 45° to the die and so makes an angle $\left(\frac{\pi}{4} - \alpha\right)$ with the axis. Thus $\frac{\pi}{4} - \zeta + \psi = \frac{\pi}{4} - \alpha$, confirming the relationship 6.34.

This solution may be checked for a simple example, where $\psi = 0$. Then $\zeta = \alpha$ and

$$q = p_F + 2k\alpha + k \tag{6.35}$$

The condition $\psi = 0$ occurs in fact at the reduction $r = 2 \sin \alpha/(1 + 2 \sin \alpha)$. Equation 6.35 gives the same value as was found in §6.5.1, by considering this reduction explicitly (equation 6.28). It is also the same value as is found for frictionless extrusion with this reduction (Chapter 8, equation 8.14), but in the extrusion example, p_F was found to be equal to k. The die pressure q is not always uniform, as is shown for heavy reductions in extrusion (§8.3.3).

Values of p_F and q were first obtained by Hill and Tupper[6.5]. The drawing stress is found by equating the drawing force to the longitudinal component of the die pressure, as in §6.5.1

$$t_x = q\frac{r}{r-1} \tag{6.29}$$

For example, if $r = 0{\cdot}1$ and $\alpha = 15°$ the predicted drawing stress is

$$\frac{t_x}{2k} = 0{\cdot}21$$

This may be compared with the homogeneous-deformation stress,

$$\frac{\sigma}{2k} = \ln 1{\cdot}11 = 0{\cdot}104$$

This is actually a larger correction than would be encountered in practical drawing conditions, because die angles steeper than 15° are unfavourable also from the point of view of lubrication. Reductions of area appreciably less than 10%, with 15° dies, are insufficient to cause plastic deformation across the whole section. The dies then act separately, and the slip-line field is more like that for indentation of a very thick block (Chapter 5, §5.6.5). The constraint is then very large, the die pressure rises, and the metal tends to be displaced laterally, causing a bulge in the surface before the strip enters the die, rather like the pile-up of metal round an indentation. The process can also be likened to machining with a very high, negative, rake angle. This bulging occurs when the die pressure reaches a value

$$q = 2k\left(1 + \frac{\pi}{2} - \alpha\right) \text{ at } r = \alpha\left(0{\cdot}23 + \frac{\alpha}{9}\right) \tag{6.36}$$

The slip-line field described above is not valid beyond the bulging limit. Bulging is observed experimentally at about the predicted stage, but it is detrimental to the lubrication and to the strip itself, and drawing should not be performed under these conditions.

work, friction and work hardening is then given by

$$t_H' = \frac{\sigma_H'}{\sigma_H} \cdot t_H \qquad (6.45)$$

The value of t_H is found from equation 6.43, assuming that the mean equivalent strain is independent of the strain-hardening characteristics, and of the magnitude of the friction coefficient.

$$t_H' = \frac{\sigma_H'}{\sigma_H} \cdot \int_0^{\epsilon_2} 2k \, \mathrm{d}\epsilon \qquad (6.46)$$

or, since

$$\epsilon_2 = f\left(\frac{c}{d}\right)\epsilon_a, \quad t_H' = f\left(\frac{c}{d}\right)\sigma_H'$$

The ratio σ_H'/σ_H can easily be found with the aid of a computer. The basic differential equation derived from stress evaluation, including friction, is given by equation 6.6.

$$\frac{\mathrm{d}\sigma_x'}{\mathrm{d}\epsilon} + B\sigma_x' = (1 + B)\, 2k \qquad (6.47\text{a})$$

In the absence of friction:

$$\frac{\mathrm{d}\sigma_x}{\mathrm{d}\epsilon} = 2k \qquad (6.47\text{b})$$

The relationship between k and ϵ is found from the stress–strain curve, and the ratio σ_x'/σ_x can be obtained by solving these equations with the computer for any given stress–strain curve.

An experimental check of the predicted drawing stresses t_H', for a wide range of reductions of area and die angles, showed good correlation. The calculations involved are all simple, though most expeditiously performed by a small computer. It appears that the necessary coefficients of friction can be determined independently, provided that care is taken to ensure similarity of lubrication conditions. The theory is thus suitable for practical application, but strip-drawing is not a common industrial procedure. Green has suggested that, with small modifications, it could be applied to drawing of thin-walled tubes, which is industrially important. (See Chapter 7.)

6.7 Determination of drawing stress for round bar, allowing for friction, redundant work and strain-hardening

There is no rigorous theory to allow for redundant work in drawing round bar, but the stress-evaluation method allows for friction and strain-hardening. Because low die angles are used industrially, and heavy reductions are commonly taken, the redundant work factor is often small in practical wire-drawing, but it cannot always be neglected. It is particularly significant in multiple light passes, since the additional hardening is cumulative.

If the material does not strain-harden, or if a mean yield stress can be used, the but it is more expensive. Zinc phosphate is the most common ingredient. In warm acid solution the following reactions are believed to occur:

$$Fe^{+++} + 2H_2PO_4 \rightarrow Fe(H_2PO_4)_2$$

$$3Fe^{++} + 3Zn^{++} + 4PO_4^{---} \rightarrow Zn_3(PO_4)_2 + Fe_3(PO_4)_2$$

Other ions such as Mn^{++++} may be present, and activators such as nitrates, nitrites or chlorates are incorporated to accelerate the attack. Temperature is important in determining the reaction rate, and pH must be controlled to avoid direct acidic corrosion. Commercial solutions are carefully prescribed.

Phosphate conversion-coatings by themselves will protect from pickup but give high friction and heavy wear. It is always necessary to use an additional lubricant, though some phosphating baths contain a lubricating ingredient. Dry calcium stearate powder is often used. The slightly-rough coated wire picks this up from a box just ahead of the die and draws the powder forward into the die mouth, where it is strongly compacted and passes through the die as a coherent thin film. The process is critical, and the soap must not be allowed to coagulate so that the wire passes through uncoated, by channelling a stable hole through the powder.

Water-soluble sodium stearate avoids this problem, but involves an additional separate sequence of washing, dipping in the soap solution, and drying. Wet soap solution is much less effective than the dry coating deposited from it. This technique is more widely used for drawing tubes, where the bore lubrication is crucial and the unit cost of the workpiece is higher.

For less severe wire-drawing operations, and for higher speeds, borax or lime coatings are used as carriers for the lubricant. Large bars, 50–100 mm in diameter are usually drawn slowly with only light sizing passes. Grease or tallow, applied at the die, gives adequate lubrication.

Wet drawing is used for moderate passes at high speeds. Soap-fat emulsions are commonly used, with a typical composition:

potassium stearate 35%, tallow 25%, mineral oil 8%, stearic acid 2%, water 30%.

Since the stability of emulsions can cause trouble, and an inversion to water-in-oil emulsion may lead to corrosion of the machine, many operators prefer compounded neat oils. These are mineral oils with fatty acid, sulphonated, chlorinated or sulphurised additives in various proportions. It is believed that these oils give better finish than the emulsions, but there is the usual balance between high surface reflectivity with thin lubricant films, and matt surfaces with thicker films giving greater protection. The more viscous oils give less shiny surfaces.

Bright-drawing to size, of angles and other sections, also uses oils. Stainless steels are more prone to pickup on the dies than other steels, for reasons that are not fully explained but may depend upon the thin natural oxide film being easily ruptured, and on the high strain-hardening rate of stainless steel. Oils containing chlorinated wax, or straight chlorinated oils containing a high weight proportion of chlorine are effective. The major constituent is a hydrocarbon containing 35–40% Cl, and care must always be taken to control the production of corrosive HCl in moist conditions.

Phosphate conversion coatings cannot be produced on the unreactive surface of