# Problem 27

A cart loaded with sand is dragged by a constant force $F\hat{x}$. Because of a hole at the bottom of the cart, sand is spilled at an even rate $\frac{dm}{dt} = -\lambda$. Write and solve the equations of motion for the $x$ axis.

## Solution:

The equation of motion is

$$F = \frac{d(mv)}{dt} = m(t)\frac{dv}{dt} \tag{27.1}$$

*Note*: The force applied to the cart by the sand *in the $x$ direction* is zero, because the $\hat{x}$ component of the velocity of the sand grains relative to the cart is zero. This is because the sand is spills perpendicularly to the direction of motion. Therefore, the only change of momentum in the $\hat{x}$ direction is due to the decrease of $M$.

By substituting $m(t)$, we obtain

$$(M_0 - \lambda t)\frac{dv}{dt} = F \tag{27.2}$$

$M_0$ is the mass of the cart, including the sand at $t = 0$. By changing the form a bit, we derive

$$dv = \frac{F}{M_0 - \lambda t}\,dt \tag{27.3}$$

Integration yields

$$v(t) = v_0 + \frac{F}{\lambda}\ln\left(\frac{M_0}{M_0 - \lambda t}\right) \tag{27.4}$$

# Problem 28

A ball with mass $\frac{M}{2}$ filled with gas (whose mass is $\frac{M}{2}$) is standing on a frictionless table. A bullet of mass $m = \frac{M}{4}$ and velocity $v_0\hat{x}$ penetrates the ball, and is rests inside at $t = 0$ (see Figure 28.1). Assume that the amount of gas emitted during the collision can be

Similarly, it can be shown that

$$zF_x - xF_z = N_y \quad (29.17)$$

$$xF_y - yF_x = N_z \quad (29.18)$$

We can further see that

$$\frac{dE_k}{dt} = 2160\cos(6t) + 1620\sin(6t) \quad (29.19)$$

and,

$$\vec{F}\cdot\vec{v} = F_x v_x + F_y v_y + F_z v_z = 2160\cos(6t) + 1620\sin(6t) = \frac{dE_k}{dt} \quad (29.20)$$

*Note*: Generally, the aim of the physicist is to find $\vec{r}(t)$ through the motion equations and by knowing the forces, torques, etc., which act on the particle. In this problem, we apply the inverse process, so do not regard this problem as a reliable representative of the ability demanded from a student, but mainly as a computational problem.

# Problem 30

A cannon is placed on an inclined plane, as shown in Figure 30.1. A projectile is fired with an initial velocity, $v_0$, forming an angle $\theta$ relative to the inclined plane.

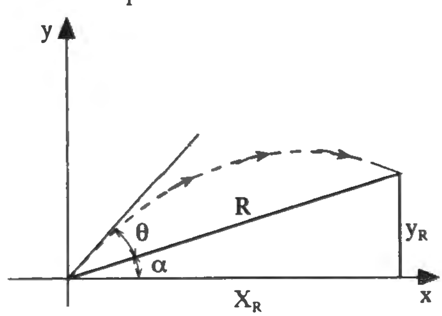


**Figure 30.1.**

1. Find the acceleration vector, the velocity vector and the position vector, as functions of time.

4. Expanding the exponent in $y(t)$ in a *Taylor series*, we obtain:

$$
\begin{aligned}
y(t) &= -\frac{m^2}{b^2} g\left[1 - \frac{b}{m}t + \frac{b^2}{2m^2}t^2 + O\left(\frac{b^3t^3}{m^3}\right) - 1\right] - \frac{mgt}{b} + h \\
&= h - \frac{gt^2}{2} + O\left(\frac{gbt^3}{m}\right)
\end{aligned}
\tag{34.9}
$$

For $t \ll \frac{m}{b}$, $\frac{gbt^3}{m} \ll gt^2$; thus a sensible approximation is

$$
y(t)_{\text{approx}} \approx h - \frac{gt^2}{2} \tag{34.10}
$$

# Problem 35

Consider a uniform and constant magnetic field, $\vec{B} = B\hat{z}$. A particle enters it at the origin at time $t = 0$ and with velocity $\vec{v}_0 = v_0(\hat{x} + \hat{z})$.

1. Write the motion equations for each coordinate and solve them.
2. Plot the path of the particle.

## Solution:

1. The force acting on the particle is the *Lorentz force*, when the electric field $\vec{E} = 0$, so that:

$$
\vec{F} = \frac{q}{c}\vec{v} \times \vec{B} = \frac{q}{c}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix} = \frac{q}{c}(v_yB, -v_xB, 0) \tag{35.1}
$$

The equation of motion are given by:

$$
\vec{a} = \frac{1}{m}\vec{F} = \frac{qB}{mc}(v_y, -v_x, 0) \tag{35.2}
$$

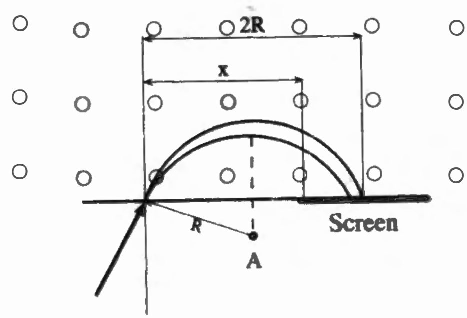


**Figure 36.2.**

*Explanation* — The significance of this result can be understood by considering a process without a magnetic field. In such a case, the distance between the points on the screen $x_0$ is:

$$x_0 \simeq 2R\theta \tag{36.7}$$

where $\theta$ is a small angle. Focusing is bringing seperate points on the screen closer to one another. According to what we have obtained in this solution, the distance between the points in the presence of a magnetic field is $R\theta^2$. We can clearly see that $R\theta^2 \ll 2R\theta$ for small angles. This phenomenon is known as a magnetic focusing.

# Problem 37

A particle of mass $m$ and charge $q$ is placed in mutually uniform perpendicular electric and magnetic fields,

$$\begin{cases} \vec{E} = (E, 0, 0) \\ \vec{B} = (0, 0, B) \end{cases}$$