Then transform the last term of this expression:

$$v_\tau \frac{d\boldsymbol{\tau}}{dt} = v_\tau \frac{d\boldsymbol{\tau}}{dl}\frac{dl}{dt} = v_\tau^2 \frac{d\boldsymbol{\tau}}{dl} = v^2 \frac{d\boldsymbol{\tau}}{dl}. \qquad (1.7)$$

Let us determine the increment of the vector $\boldsymbol{\tau}$ in the interval dl (Fig. 4). It can be strictly shown that when point *2* approaches point *1*, the segment of the path between them tends to turn into an arc of a circle with centre at some

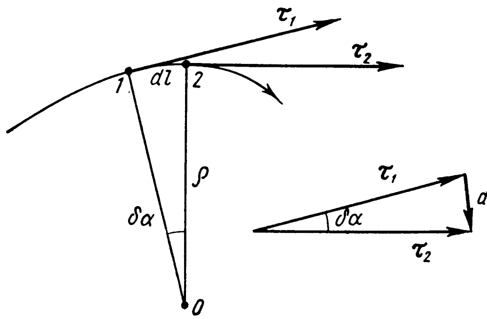

Fig. 4

point O. The point O is referred to as the centre of curvature of the path at the given point, and the radius ρ of the corresponding circle as the radius of curvature of the path at the same point.

It is seen from Fig. 4 that the angle $\delta\alpha = |\, dl\, |/\rho = |\, d\boldsymbol{\tau}\, |/1$, whence

$$|\, d\boldsymbol{\tau}/dl\, | = 1/\rho;$$

at the same time, if $dl \to 0$, then $d\boldsymbol{\tau} \perp \boldsymbol{\tau}$. Introducing a unit vector $\mathbf{n}$ of the normal to the path at point *1* directed toward the centre of curvature, we write the last equality in a vector form:

$$d\boldsymbol{\tau}/dl = \mathbf{n}/\rho. \qquad (1.8)$$

Now let us substitute Eq. (1.8) into Eq. (1.7) and then the expression obtained into Eq. (1.6). Finally we get

$$\boxed{\mathbf{w} = \frac{dv_\tau}{dt}\boldsymbol{\tau} + \frac{v^2}{\rho}\mathbf{n}.} \qquad (1.9)$$

Generally speaking, the position of the instantaneous axis varies with time. For example, in the case of a cylinder rolling over a plane surface the instantaneous axis coincides at any moment with the line of contact between the cylinder and the plane.

Angular velocity summation. Let us analyse the motion of a solid rotating simultaneously about two intersecting axes. We shall set into rotation a certain solid at the angular velocity ω' about the axis OA (Fig. 12), and then we shall set this axis into rotation with the angular velocity ω_0 about the axis OB which is stationary in the K reference frame. Let us find the resultant motion in the K frame.

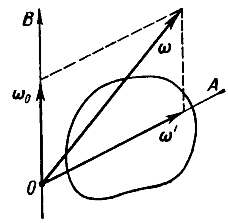

Fig. 12

We shall introduce an auxiliary reference frame K' fixed rigidly to the axes OA and OB. It is clear that this frame rotates with the angular velocity ω_0 while the solid rotates relative to this frame with the angular velocity ω'.

During the time interval dt the solid will turn through an angle $d\varphi'$ about the axis OA in the K' frame and simultaneously through $d\varphi_0$ about the axis OB together with the K' frame. The cumulative rotation follows from Eq. (1.12): $d\varphi = d\varphi_0 + d\varphi'$. Dividing both sides of this equality by dt, we obtain

$$\omega = \omega_0 + \omega'. \tag{1.20}$$

Thus, the resultant motion of the solid in the K frame is a pure rotation with the angular velocity ω about an axis coinciding at each moment with the vector ω and passing through the point O (Fig. 12). This axis is displaced relative to the K frame: it rotates together with the OA axis about the axis OB at the angular velocity ω_0.

It is not difficult to infer that even when the angular velocities ω' and ω_0 do not change their magnitudes, the body in the K frame will possess the angular acceleration β directed, according to Eq. (1.14), beyond the plane (Fig. 12). The angular acceleration of a solid is analysed in detail in Problem 1.10.

Then transform the last term of this expression:

$$v_\tau \frac{d\boldsymbol{\tau}}{dt} = v_\tau \frac{d\boldsymbol{\tau}}{dl} \frac{dl}{dt} = v_\tau^2 \frac{d\boldsymbol{\tau}}{dl} = v^2 \frac{d\boldsymbol{\tau}}{dl}. \qquad (1.7)$$

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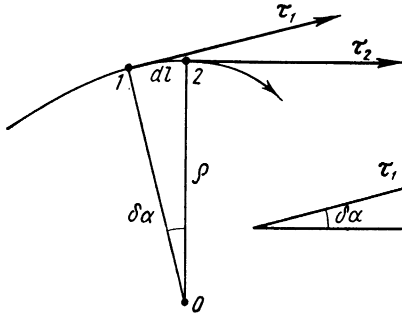

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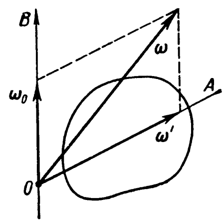

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CONTENTS

PART TWO
RELATIVISTIC MECHANICS