

Then transform the last term of this expression:

$$v_{\tau} \frac{d\tau}{dt} = v_{\tau} \frac{d\tau}{dl} \frac{dl}{dt} = v_{\tau}^2 \frac{d\tau}{dl} = v^2 \frac{d\tau}{dl}. \quad (1.7)$$

Let us determine the increment of the vector τ in the interval dl (Fig. 4). It can be strictly shown that when point 2 approaches point 1, the segment of the path between them tends to turn into an arc of a circle with centre at some

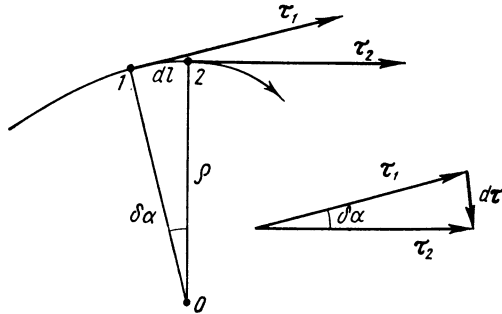


Fig. 4

point O . The point O is referred to as the centre of curvature of the path at the given point, and the radius ρ of the corresponding circle as the radius of curvature of the path at the same point.

It is seen from Fig. 4 that the angle $\delta\alpha = |dl|/\rho = |d\tau|/1$, whence

$$|d\tau/dl| = 1/\rho;$$

at the same time, if $dl \rightarrow 0$, then $d\tau \perp \tau$. Introducing a unit vector \mathbf{n} of the normal to the path at point 1 directed toward the centre of curvature, we write the last equality in a vector form:

$$d\tau/dl = \mathbf{n}/\rho. \quad (1.8)$$

Now let us substitute Eq. (1.8) into Eq. (1.7) and then the expression obtained into Eq. (1.6). Finally we get

$$\boxed{\mathbf{w} = \frac{dv_{\tau}}{dt} \tau + \frac{v^2}{\rho} \mathbf{n}.} \quad (1.9)$$

Generally speaking, the position of the instantaneous axis varies with time. For example, in the case of a cylinder rolling over a plane surface the instantaneous axis coincides at any moment with the line of contact between the cylinder and the plane.

Angular velocity summation. Let us analyse the motion of a solid rotating simultaneously about two intersecting axes. We shall set into rotation a certain solid at the angular velocity ω' about the axis OA (Fig. 12), and then we shall set this axis into rotation with the angular velocity ω_0 about the axis OB which is stationary in the K reference frame. Let us find the resultant motion in the K frame.

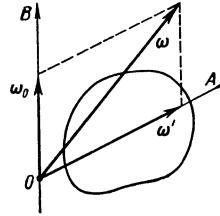


Fig. 12

We shall introduce an auxiliary reference frame K' fixed rigidly to the axes OA and OB . It is clear that this frame rotates with the angular velocity ω_0 while the solid rotates relative to this frame with the angular velocity ω' .

During the time interval dt the solid will turn through an angle $d\varphi'$ about the axis OA in the K' frame and simultaneously through $d\varphi_0$ about the axis OB together with the K' frame. The cumulative rotation follows from Eq. (1.12): $d\varphi = d\varphi_0 + d\varphi'$. Dividing both sides of this equality by dt , we obtain

$$\omega = \omega_0 + \omega'. \quad (1.20)$$

Thus, the resultant motion of the solid in the K frame is a pure rotation with the angular velocity ω about an axis coinciding at each moment with the vector ω and passing through the point O (Fig. 12). This axis is displaced relative to the K frame: it rotates together with the OA axis about the axis OB at the angular velocity ω_0 .

It is not difficult to infer that even when the angular velocities ω' and ω_0 do not change their magnitudes, the body in the K frame will possess the angular acceleration β directed, according to Eq. (1.14), beyond the plane (Fig. 12). The angular acceleration of a solid is analysed in detail in Problem 1.10.

Then transform the last term of this expression:

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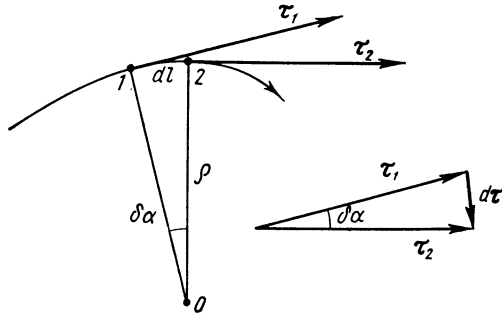


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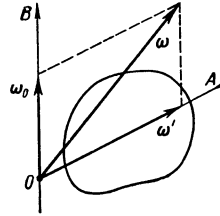


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