range and the proportional limit is as high as 25×10^3–30×10^3 lb per sq in. For materials such as cast iron or soft copper the proportional limit is very low, i.e., deviations from Hooke's law may be noticed at a low tensile stress.

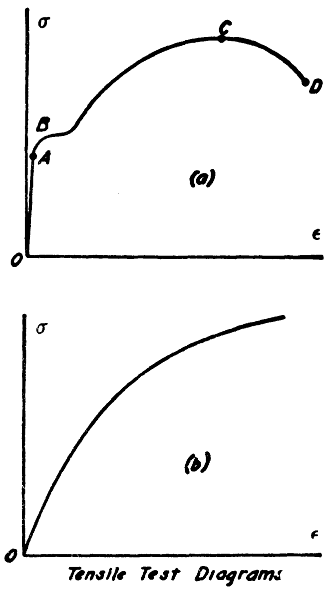

FIG. 4.

In investigating the mechanical properties of materials beyond the proportional limit, the relationship between the strain and the corresponding stress is usually presented graphically by a *tensile test diagram.* Fig. 4*a* presents a typical diagram for structural steel. Here the elongations are plotted along the horizontal axis and the corresponding stresses are given by the ordinates of the curve *OABCD*. From *O* to *A* the stress and the strain are proportional; beyond *A* the deviation from Hooke's law becomes marked; hence the stress at *A* is the *proportional limit.* Upon loading beyond this limit the elongation increases more rapidly and the diagram becomes curved. At *B* a sudden elongation of the bar takes place without an appreciable increase in the tensile force. This phenomenon, called *yielding* of the metal, is shown in the diagram by an almost horizontal portion of the curve. The stress corresponding to the point *B* is called the *yield point.* Upon further stretching of the bar, the material recovers its resistance and, as is seen from the diagram, the tensile force increases with the elongation up to the point *C*, where the force attains its maximum value. The corresponding stress is called the *ultimate strength* of the material. Beyond the point *C*, elongation of the bar takes place with a diminution of the load and fracture finally occurs at a load corresponding to point *D* of the diagram.

It should be noted that stretching of the bar is accompanied by lateral contraction but it is established practice in calculat-

$$\sqrt{a^2(1 + 2\epsilon_x) + b^2(1 + 2\epsilon_y)} \approx \sqrt{a^2 + b^2}\left(1 + \frac{a^2\epsilon_x}{a^2 + b^2} + \frac{b^2\epsilon_y}{a^2 + b^2}\right).$$

Subtracting from this the initial length $\sqrt{a^2 + b^2}$ and dividing by the initial length, we obtain

$$\epsilon = \epsilon_x \cos^2 \alpha + \epsilon_y \sin^2 \alpha.$$[9]

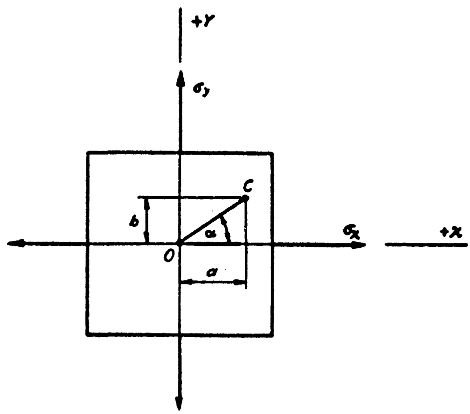

Fig. 44.

16. Pure Shear.—*Modulus in Shear.*—Let us consider the particular case of normal stresses acting in two perpendicular directions in which the tensile stress σ_x in the horizontal direction is numerically equal to the compressive stress σ_y in the vertical direction, Fig. 45*a*. The corresponding circle of stress is shown in Fig. 45*b*. Point D on this circle represents the stresses acting on the planes *ab* and *cd* perpendicular to the *xy* plane and inclined at 45° to the x axis. Point D_1 represents stresses acting on the planes *ad* and *bc* perpendicular to *ab* and *cd*. It is seen from the circle of stress that the normal stress on each of these planes is zero and that the shearing stress over these planes, represented by the radius of the circle, is numerically equal to the normal stress σ_x, so that

$$\tau = \sigma_x = -\sigma_y. \qquad (a)$$

[9] This equation is similar to eq. (26). Thus, a graphical representation of strain (strain circle), similar to Mohr's circle for stress, can be used.

of the beam and taking $x < a_1$, we obtain for the first portion of the beam

$$V = R_1 \quad \text{and} \quad M = R_1 x. \qquad (e)$$

For the second portion of the beam, i.e., for $a_1 < x < a_2$, we obtain

$$V = R_1 - P_1 \quad \text{and} \quad M = R_1 x - P_1(x - a_1). \quad (f)$$

For the third portion of the beam, i.e., for $a_2 < x < a_3$, it is advantageous to consider the right portion of the beam rather than the left. In this way we obtain

$$V = -(R_2 - P_3)$$

and

$$M = R_2(l - x) - P_3(l - x - b_3). \qquad (g)$$

Finally for the last portion of the beam we obtain

$$V = -R_2, \qquad M = R_2(l - x). \qquad (h)$$

From expressions (*e*)–(*h*) we see that in each portion of the beam the shearing force remains constant. Hence the shearing force diagram is as shown in Fig. 67*b*. The bending moment in each portion of the beam is a linear function of x. Hence in the corresponding diagram it is represented by an inclined straight line. To draw these lines we note from expressions (*e*) and (*h*) that at the ends $x = 0$ and $x = l$ the moments are zero. The moments under the loads are obtained by substituting in expressions (*e*), (*f*) and (*h*), $x = a_1$, $x = a_2$ and $x = a_3$, respectively. In this manner we obtain for the above-mentioned moments the values

$$M = R_1 a_1, \qquad M = R_1 a_2 - P_1(a_2 - a_1), \qquad M = R_2 b_3.$$

By using these values the bending moment diagram, shown in Fig. 67*c*, is readily constructed.

In practical applications it is of importance to find the cross sections at which the bending moment has its maximum or minimum values. In the case of concentrated loads just considered, Fig. 67, the maximum bending moment occurs under the load P_2. This load corresponds in the bending moment diagram to point d_1, at which point the slope of the diagram

5. Determine the ratio of the weights of three beams of the same length under the same M and $(\sigma_x)_{max}$ and having as cross sections, respectively, a circle, a square and a rectangle with proportions $h = 2b$.

Answer. 1.12:1:0.793.

6. Make a comparison of the section moduli for two beams of the same weight if the first beam is a solid circular beam of diameter d and the second is a circular tube of outer diameter D and inner diameter D_1.

Solution. The cross-sectional area of both beams is $A = \frac{\pi d^2}{4} = \frac{\pi(D^2 - D_1^2)}{4}$. For the solid beam $Z = Ad/8$, for the tubular beam $Z_1 = \frac{\pi(D^4 - D_1^4)}{32D} = \frac{AD}{8}\left(1 + \frac{D_1^2}{D^2}\right)$. Observing that $D_1^2 = D^2 - \frac{4A}{\pi}$, we find for the tubular beam $Z_1 = \frac{AD}{8}\left(2 - \frac{4A}{\pi D^2}\right)$, so that

$$\frac{Z_1}{Z} = \frac{D}{d}\left(2 - \frac{4A}{\pi D^2}\right).$$

Thus, for very thick tubes D approaches d and Z_1 approaches Z. For very thin tubes D is large in comparison with d and the ratio $Z_1:Z$ approaches the value $2D/d$.

25. General Case of Laterally Loaded Symmetrical Beams.—In the general case of beams laterally loaded in a plane of symmetry, the stresses distributed over a cross section of a beam must balance the shearing force and the bending moment at that cross section. The calculation of the stresses is usually made in two steps by determining first the stresses produced by the bending moment, called the *bending stresses*, and afterwards the *shearing stresses* produced by the shearing force. In this article we shall limit ourself to the calculation of the bending stresses; the discussion of shearing stresses will be given in the next article. In calculating bending stresses we assume that these stresses are distributed in the same manner as in the case of pure bending and the formulas for the stresses derived in Art. 23 will be valid. (A more complete discussion of stresses near the points of application of concentrated forces is given in Part II.)

of this strip about Cz is ydA and the total moment for the entire segment is

$$\int_{y_1}^{R} 2\sqrt{R^2 - y^2}\cdot y dy = \tfrac{2}{3}(R^2 - y_1^2)^{3/2}.$$

Substituting this into eq. (64) and taking $2\sqrt{R^2 - y_1^2}$ for b, we obtain for the vertical shearing stress component

$$\tau_{xy} = \frac{V(R^2 - y_1^2)}{3I_z}, \tag{67}$$

and the total shearing stress at points p (Fig. 109) is

$$\tau = \frac{\tau_{xy}\cdot R}{\sqrt{R^2 - y_1^2}} = \frac{VR\sqrt{R^2 - y_1^2}}{3I_z}.$$

It is seen that the maximum τ is obtained for $y_1 = 0$, i.e., for the neutral axis of the cross section. Then, substituting $I_z = \pi R^4/4$, we obtain

$$\tau_{max} = \frac{4}{3}\frac{V}{\pi R^2} = \frac{4}{3}\cdot\frac{V}{A}. \tag{68}$$

In the case of a circular cross section, therefore, the maximum shearing stress is 33 per cent larger than the average value obtained by dividing the shearing force by the cross-sectional area.

28. Shearing Stresses in I Beams.—In considering the distribution of shearing stresses in the web of an I beam or WF beam (Fig. 110), the same assumptions are made as for a rectangular cross section; these were that the shearing stresses are parallel to the shearing force V and are uniformly distributed over the thickness b_1 of the web. Then eq. (64) may be used for calculating the stresses τ_{xy}. For points on the line pp at a distance y_1 from the neutral axis, where the width of the cross section is b_1, the moment of the shaded portion of the cross section with respect to the neutral axis z is

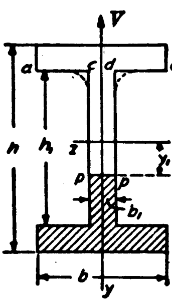

FIG. 110.

$$\int_{y_1}^{h/2} ydA = \frac{b}{2}\left(\frac{h^2}{4} - \frac{h_1^2}{4}\right) + \frac{b_1}{2}\left(\frac{h_1^2}{4} - y_1^2\right).$$

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