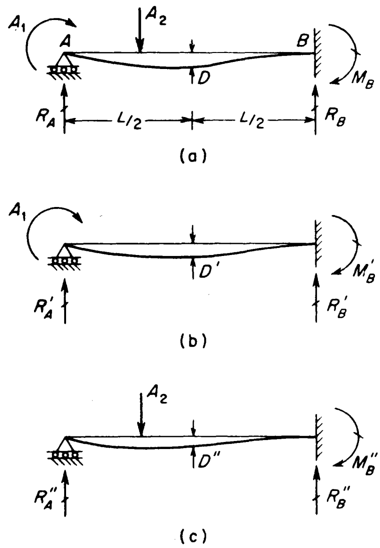


Fig. 1-14. Effects of actions.

rately are shown in Figs. 1-14b and 1-14c. In each case there is a displacement at the midpoint of the beam and reactions at the ends. A single prime is used to denote quantities associated with the action A_1, and a double prime is used for quantities associated with A_2.

According to the principle of superposition, the actions and displacements caused by A_1 and A_2 acting separately (Figs. 1-14b and 1-14c) can be combined in order to obtain the actions and displacements caused by A_1 and A_2 acting simultaneously (Fig. 1-14a). Thus, the following *equations of superposition* can be written for the beam in Fig. 1-14:

$$R_A = R'_A + R''_A \qquad R_B = R'_B + R''_B$$
$$M_B = M'_B + M''_B \qquad D = D' + D'' \tag{1-3}$$

Of course, similar equations of superposition can be written for other actions and displacements in the beam, such as stress resultants at any cross section of the beam and displacements (translations and rotations) at any point along the axis of the beam. This manner of using superposition was illustrated previously in Art. 1.4.

A second example of the principle of superposition, in which displacements are the cause, is given in Fig. 1-15. The figure portrays again the beam AB with one end simply supported and the other fixed. When end B of the beam is displaced downward through a distance Δ and, at the same time, is caused to be rotated through an angle θ (see Fig. 1-15a), various actions and displacements in the beam will be developed. For example, the reactions at each end and the displacement at the center are shown in Fig. 1-15a. The next two sketches (Figs. 1-15b and 1-15c) show the beam with

To show the use of the matrix equations given above, consider again the beam in Fig. 2-2a. In order to have a specific example, assume that the beam has constant flexural rigidity EI in both spans and that the actions on the beam are as follows:

$$P_1 = 2P \qquad M = PL \qquad P_2 = P \qquad P_3 = P$$

Also, assume that there are no support displacements at any of the supports of the structure.

The matrices to be found first in the analysis are $\mathbf{D}_\mathrm{Q}$, $\mathbf{D}_\mathrm{QL}$, and $\mathbf{F}$, as mentioned previously. Since in the original beam there are no displacements corresponding to Q_1 and Q_2, the matrix $\mathbf{D}_\mathrm{Q}$ is a null matrix. The matrix $\mathbf{D}_\mathrm{QL}$ represents the displacements in the released structure corresponding to the redundants and caused by the loads. These displacements are found by considering Fig. 2-2f, which shows the released structure under the action of the loads. The displacements in this beam corresponding to Q_1 and Q_2 can be found by the methods described in Appendix A (see Example 3, Art. A.2), and the results are:

$$D_{QL1} = \frac{13PL^3}{24EI} \qquad D_{QL2} = \frac{97PL^3}{48EI}$$

The positive signs in these expressions show that both displacements are upward. From the results given above, the vector $\mathbf{D}_\mathrm{QL}$ is obtained:

$$\mathbf{D}_\mathrm{QL} = \frac{PL^3}{48EI}\begin{bmatrix} 26 \\ 97 \end{bmatrix}$$

The flexibility matrix $\mathbf{F}$ is found by referring to the beams pictured in Figs. 2-2g and 2-2h. The beam in Fig. 2-2g, which is subjected to a unit load corresponding to Q_1, has displacements given by the expressions

$$F_{11} = \frac{L^3}{3EI} \qquad F_{21} = \frac{5L^3}{6EI}$$

Similarly, the displacements in the beam of Fig. 2-2h are

$$F_{12} = \frac{5L^3}{6EI} \qquad F_{22} = \frac{8L^3}{3EI}$$

From the results listed above, the flexibility matrix can be formed:

$$\mathbf{F} = \frac{L^3}{6EI}\begin{bmatrix} 2 & 5 \\ 5 & 16 \end{bmatrix}$$

The inverse of the flexibility matrix can be found by any one of several standard methods,* the result being

*See, for instance, J. M. Gere and W. Weaver, Jr., *Matrix Algebra for Engineers,* D. Van Nostrand, New York, 1965

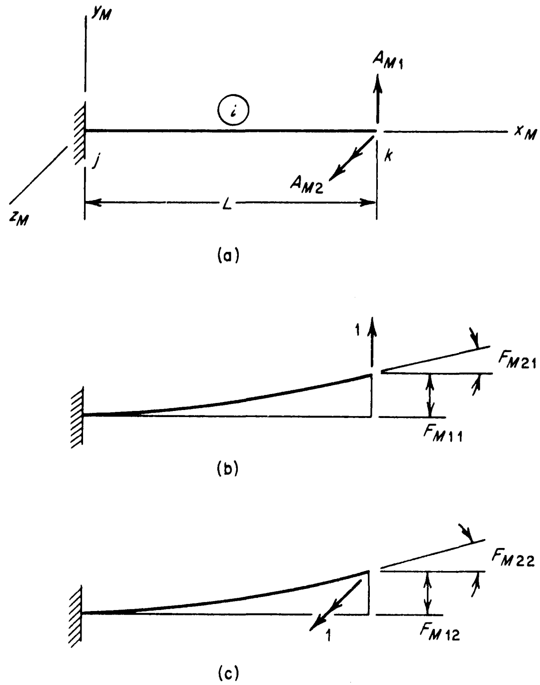


Fig. 2-13. Flexibilities for beam member.

the beam member shown in Fig. 2-13a. In this case the y_M axis is chosen so that bending takes place in the x_M-y_M plane (a principal plane of bending). Two kinds of end-actions are indicated at the k end of the member in Fig. 2-13a: a shearing force A_{M1} (positive in the y_M direction) and a bending moment A_{M2} (positive in the z_M sense). The flexibility matrix of interest here is a 2 × 2 array relating A_{M1} and A_{M2} to the corresponding displacements D_{M1} (translation in the y_M direction) and D_{M2} (rotation in the z_M sense). Figures 2-13b and 2-13c show the application of unit loads $A_{M1} = 1$ and $A_{M2} = 1$ to obtain the terms in the beam flexibility matrix $\mathbf{F}_{Mi}$, as follows:

$$\mathbf{F}_{Mi} = \begin{bmatrix} F_{M11} & F_{M12} \\ F_{M21} & F_{M22} \end{bmatrix} = \begin{bmatrix} \dfrac{L^3}{3EI} & \dfrac{L^2}{2EI} \\ \dfrac{L^2}{2EI} & \dfrac{L}{EI} \end{bmatrix} \tag{2-19}$$

These terms were found previously in the example at the end of Art. 1.11.

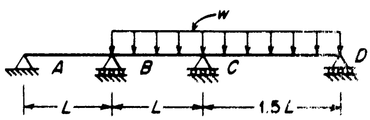
The truss member in Fig. 2-14a has only one end-action to be considered for the purpose of calculating member flexibilities, namely, the axial shown in Fig. 2-3a, due to the force P and moment M acting at the middle of the span. The beam has constant flexural rigidity EI and length L. Select the reactive moments themselves as the redundant actions, and assume these moments are positive when they produce compression on the bottom of the beam. Take the first redundant at end A of the beam and the second at end B.

2.3-2. Analyze the two-span beam shown in Fig. 2-2a by taking the reactive moment at support A and the bending moment just to the left of support B as the redundants Q_1 and Q_2, respectively. Assume that these moments are positive when they produce compression on the top of the beam. Also, assume that the loads on the beam are $P_1 = 2P$, $M = PL$, $P_2 = P$, $P_3 = P$, and the flexural rigidity EI is constant.

2.3-3. Analyze the two-span beam of Fig. 2-2a if support B is displaced downward by a small distance s. Select the redundants to be the vertical reactions at supports B and C, as shown in Fig. 2-2a, and omit the effects of the loads in the analysis. Assume that EI is constant for both spans.

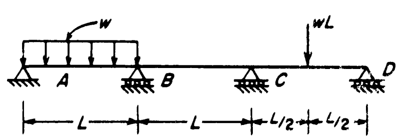
2.3-4. Find the redundant actions for the two-span beam of Fig. 2-2a using the released structure shown in Fig. 2-2b. Assume that EI is constant for the beam and that the loads are $P_1 = P$, $M = 0$, $P_2 = P$, $P_3 = P$. Number the redundants from left to right along the beam; also, assume that the redundant moment is positive when counterclockwise, and that the redundant force is positive when upward.

2.3-5. Determine the bending moments at supports B and C of the continuous beam shown in the figure, using these moments as the redundants Q_1 and Q_2, respectively. Assume that the redundants are positive when they produce compression on the top of the beam. The beam has constant flexural rigidity EI.



Prob. 2.3-5.

2.3-6. Find the bending moments at supports B and C of the continuous beam (see figure), using these moments as the redundants Q_1 and Q_2, respectively. Assume that Q_1 and Q_2 are positive when they produce compression on the top of the beam. The flexural rigidity of the beam is EI.



Prob. 2.3-6.

2.3-7. Analyze the plane truss shown in Fig. 2-5a by taking the forces in the two diagonal members AD and BC as the redundants Q_1 and Q_2, respectively. Assume that tension in a member is positive, and assume that there are no support displacements. All members have the same axial rigidity EA.

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