Contents

Chapt	er 1 Introduction	1–62
1.1	Introduction	1
1.2	Number System	1
1.3	Interval	2
1.4	Concept of Sets	3
1.5	Type of Sets	6
1.6	Subset	7
1.7	Universal Set	9
1.8	Venn Diagrams	13
1.9	Operations on Sets	13
1.10	Some Results on Venn Diagram	21
1.11	Ordered Pair	23
1.12	Relation	28
1.13	Classification of Relations	30
1.14	Functions	39
1.15	Type of Functions	40
1.16	Binary Operation	47
1.17	Number of Binary Operations	47
1.18	Properties of Binary Operation	47
1.19	Algebraic Structure	48
1.20	Identity Element	52
1.21	Inverse of an Element	53
1.22	Composition Table for Binary Operation on Finite Sets	55
Chapt	er 2 Matrix Algebra	63–136

Chapter 2 Matrix Algebra

2.1	Introduction	63
2.2	Type of Matrices	63
2.3	Operation on Matrices	65
2.4	Properties of Matrix Addition	67
2.5	Properties of Multiplication of Matrix by a Scalar	68
2.6	Multiplication of Matrices	72
2.7	Properties of Matrix Multiplication	73
2.8	Determinant of a Square Matrix	75
2.9	Properties of Determinants	76
2.10	Evolution of a Determinant by Sarrus Diagram	76
2.11	Minor and Cofactors	77
2.12	Singular and Non-Singular Matrix	77
2.13	Transpose of a Matrix	78
2.14	Properties of Transpose of a Matrix	78
2.15	Symmetric Matrix	79
2.16	Skew-Symmetric Matrix	79
2.17	Properties of Symmetric and Skew-Symmetric Matrix	80
2.18	Complex Matrix	81
2.19	Submatrix of a Matrix	86
2.20	Minors of a Matrix	87
2.21	Rank of a Matrix	87
2.22	Echleon From of a Matrix	88

2.23 2.24 2.25 2.26 2.27 2.28 2.29 2.30	Elementary Transformations (or E-Transformations) of a Matrix Elementary Matrices Invariance of Rank under E-Transformations Normal Form Equivalence of Matrices Row and Column Equivalence of Matrices Inverse of a Matrix Inverse of a Matrix by Elementary Transformations	92 93 95 96 98 99 116 121
Chapt	er 3 System of Linear Equations	137–176
3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8	Introduction Vectors and their Dependence and Independence Homogeneous Linear Equations Nature of the Solution of the Equation AX = 0 Non-Homogeneous Equations Condition for Consistency Gauss Elimination Method LU Decomposition Method	137 137 140 142 148 148 148 155 163
Chapt	er 4 Eigen Values and Eigen Vectors of a Matrix	177–234
4.1 4.2 4.3 4.4 4.5 4.6 4.7 4.8	Introduction The Characteristic Equation of a Matrix Characteristic Vectors or Eigenvectors of a Matrix Relation Between Eigenvalues and Eigenvectors Eigenvalues of Special Type of Matrices The Cayley-Hamilton Theorem Diagonalisation of a Matrix Algebraic and Geometric Multiplicity of an Eigenvalue	177 177 178 178 180 202 214 222
Chapt	er 5 Vector Spaces	235–348
$5.1 \\ 5.2 \\ 5.3 \\ 5.4 \\ 5.5 \\ 5.6 \\ 5.7 \\ 5.8 \\ 5.9 \\ 5.10 \\ 5.11 \\ 5.12 \\ 5.13 \\ 5.14 \\ 5.15 \\ 5.16 \\ 5.17 $	Introduction Prologue to Vector Space Vector Spaces Elementary Properties of Vector Spaces Vector Subspaces: Vector Space within Vector Space Elementary Properties of Vector Subspaces Algebra of Subspaces Linear Sum of Two Subspaces Direct Sum of Vector Subspaces Linear Combination of Vectors Linear Dependence and Independence of Vectors Basis of a Vector Space Finite Dimensional Vector Space Dimension of Subspace of a Vector Space Cosets Quotient Spaces Isomorphism	235 235 237 251 253 253 254 264 265 273 275 290 290 290 290 294 317 318 321
Chapt	er 6 Linear Transformations	349–418
6.1 6.2 6.3 6.4	Introduction Some Definitions Properties of Linear Transformations Algebra of Linear Transformations	349 350 350 352

6.4 Algebra of Linear Transformations

6.5	Linear Operator	356
6.6	Algebra of Linear Operators	356
6.7	Range and Null Space of a Linear Transformation	357
6.8	Product of Linear Transformations	358
6.9	Polynomials in a Linear Operator	360
6.10	Invertible Linear Transformation	361
6.11	Non-Singular Linear Transformations	362
6.12	Coordinate Vector	370
6.13	Matrix Representation of a Linear Transformation	370
6.14	Change of Basis	375
6.15	Similarity of Matrices	377
6.16	Similarity of Linear Transformations	378
6.17	Determinant of a Linear Transformation on a Finite Dimensional Vector Space	381
6.18	Scalar Transformation	381
6.19	Trace of a Matrix	381
6.20	Trace of a Linear Transformation on a Finite Dimensional Vector Space	382
	*	

0.20		
Chapte	er 7 Linear Functionals	419–470
7.1	Introduction	419
7.2	Dual Spaces	420
7.3	Dual Basis	423
7.4	Second Dual Space: Bidual Space	426
7.5	Natural Mapping	429
7.6	Annihilator	429
7.7	Annihilator of an Annihilator	431
7.8	Eigenvalues and Eigenvectors of a Linear Transformation	445
7.9	Minimal Polynomial	448
7.10	Invariance of Linear Operator	449
7.11	Diagonalization	453

471–554

8.1	Introduction	471
8.2	Orthogonality and Orthonormality	474
8.3	Orthogonal Expansion	480
8.4	The Adjoint of a Linear Transformation	489
8.5	Properties of the Adjoint	490
8.6	Self-Adjoint Transformation	491
8.7	Congruent Operators	498
8.8	Inner Product Vector Space Isomorphism	499
8.9	Unitary Operators	502
8.10	Normal Operators	507
8.11	Positive Operators	509
8.12	Perpendicular Projection	514
8.13	Properties of a Perpendicular Projection	516
8.14	Invariance and Reducibility in Inner Product Space	518
8.15	Orthogonal Projections	521
8.16	Characterization of Spectra	526
8.17	The Spectral Theorem for Normal Operators	530
8.18	Spectral Theorem for Self-Adjoint Operators	535

Chapt	er 9 Bilinear, Quadratic and Hermitian Forms	555–644
9.1	Introduction	555
9.2	Bilinear Forms	555
9.3	Bilinear Forms and Matrices	557

9.4	Quadratic Forms	575
9.5	Real Symmetric Bilinear and Quadratic Forms: Law of Inertia	576
9.6	Orthogonal Diagonalization of the Quadratic Form	578
9.7	Quadratic Forms and Matrices	585
9.8	Matrix of Quadratic Form	586
9.9	Conversion of a Symmetric Matrix into Quadratic Form	587
9.10	Congruence Operation on a Square Matrix	593
9.11	Congruence of Quadratic Forms	593
9.12	Equivalence of Real Quadratic Forms	593
9.13	The Linear Transformation of a Quadratic Form	594
9.14	Congruent Reduction of a Symmetric Matrix	594
9.15	Rank of a Quadratic Form	596
9.16	Reduction of a Real Quadratic Form over Real Field	599
9.17	Normal (or Canonical) Form of a Real Quadratic Matrix	600
9.18	Signature and Index of a Real Quadratic Form	602
9.19	Reduction of a Real Quadratic Form over the Field of Complex Numbers	603
9.20	Orthogonal Reduction of a Real Quadratic Form	604
9.21	Classification of Real Symmetric Matrices	616
9.22	Positive-Definiteness of a Quadratic Form X'AX in Terms of Leading Principal Minors of A	622
9.23	Hermitian Forms	634
9.24	Matrix Representation of a Hermitian Form	635
hapt	er 10 Canonical Forms 645–6	668

Chapter 10 Canonical Forms

10.1	Introduction	645
10.2	Similarity of Linear Transformations	645
10.3	Invariant Subspace	645
10.4	Invariant Direct-Sum Decompositions	646
10.5	Normal Form	647
10.6	Triangular Form	648
10.7	Nilpotent Transformation	651
10.8	Jordan Canonical Form	651
10.9	Rational Canonical Form	653
10.10	Raw and Column Space of a Matrix	653

Chapter 11 Modules

Chapte	r 11 Modules	669–705
11.1	Introduction	669
11.2	Modules	669
11.3	Coset R-Module	670
11.4	General Properties of Modules	670
11.5	Submodules	670
11.6	Linear Sum of Two Modules	672
11.7	Homomorphism of Modules (Linear Transformations)	673
11.8	Quotient Modules	675
11.9	Cyclic Module	675
11.10	Simple and Semi-Simple Modules	678
11.11	Free Modules	681
11.12	Noetherian and Artinian Modules	685
11.13	Filtered and Graded Modules	690
11.14	Smith Normal Form over a PID and Rank	693
11.15	Finitely Generated Modules over a PID	697
Bibliogra	phy	707
Index		709–712