

# Engineering Mathematics

with Applications

is written for mathematics courses in engineering. Core concepts are presented through a large number of solved examples followed by unsolved exercise. This text deals with the basic applications in a simple and supported by a plethora of examples, practice exercises for self-evaluation and detailed theory. Also, the text has been supplemented through remarks and notes to clarify principality of the subject.

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- ✓ Algebra
- ✓ Differential Calculus-I
- ✓ Differential Calculus-II
- ✓ Integral Calculus-I
- ✓ Integral Calculus-II
- ✓ Vector Calculus
- ✓ Series
- ✓ Differential Equations and Integral Transform Methods
- ✓ Statistical Methods
- ✓ Numerical Methods
- ✓ Complex Numbers
- ✓ Appendix

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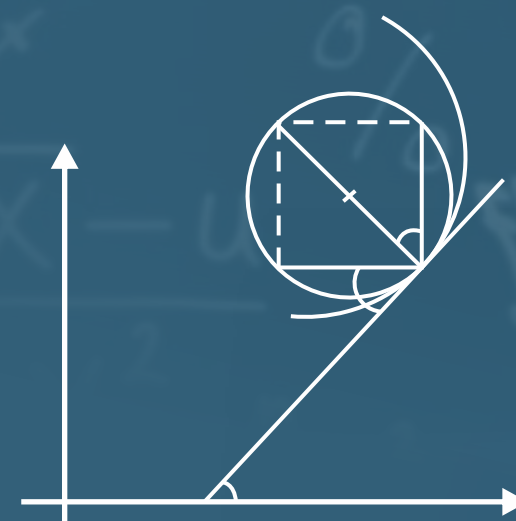
Engineering  
Mathematics  
with Applications

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*Useful for*

■ BE/BTech and all other Engineering Courses



**Sudhir Kumar Pundir**



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# ENGINEERING MATHEMATICS

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Useful for

■ BE/B Tech and other Engineering courses

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# ENGINEERING MATHEMATICS

with APPLICATIONS

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# Preface

The book entitled '**ENGINEERING MATHEMATICS with Applications**' is meant for the students of B.E., B.Tech. and all other engineering courses. The book covers almost the entire syllabi of various technical universities in India.

In this book special and conscious efforts have been made to keep the writing style simple. It is a collection and compilation work from various sources and has been endeavoured to include as much as information as could be possible. Different concepts have been explained with the help of detailed theory and examples. There is a plenty of scope in the form of exercise for the readers to try and solve the problem on his own.

I express my gratitude to the authors and publishers of various books, I consulted during the preparation of the book.

I wish sincerely thank **Sh. S.K. Jain** and **Sh. Varun Jain**, Managing directors, CBS publishers and Distributors, New Delhi for their encouragement and help in bringing out this publication in a present nice form.

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I must also record my appreciation due to my wife **Dr. Rimple**, daughter **Rijuta** and son **Shrish** for their understanding and love during the long period that I have taken to complete this book. I am indebted to my colleagues and research students who generously shared their views on the need of a comprehensive book on ENGINEERING MATHEMATICS.

Above all I am thankful to 'The Almighty God' without whose grace nothing is possible for anyone.

Further suggestions and comments for improvement of the book will be thankfully received and duly incorporated in the next edition.

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